Now when the perpendicular from the right angle to the hypotenuse is 12 the hypotenuse must be 26. Hence, substituting in the last equation we have $dy=\frac{6\times26\times\frac{1}{12}}{13}=1$; *i. e.*, the area, at the time mentioned, is increasing at the rate of 1 square inch a second.

Also solved by *L. B. FILLMAN*, *ALOIS F. KOVARIK*, *J. SCHEFFER*, *C. D. SCHMITT*, and *G. B. M. ZERR*.

109. Proposed by M. E. GRABER, Heidelberg University, Tiffin, Ohio.

Find the curve in which the product of the perpendiculars drawn from two fixed points to any tangent is constant.

Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn., and the PROPOSER.

Let the equation of the tangent be

$$y-y'=\frac{dy'}{dx'}(x-x').$$

And let the two points be $(a, 0)$ and $(-a, 0)$.

The product of the two perpendiculars is easily found to be

$$\frac{(xdy-ydx)^2-(ady)^2}{(dx)^2+(dy)^2}\text{ which}=b^2,$$

$$\text{or }\frac{(xp-y)^2-(ap)^2}{1+p^2}=b^2,\text{ or }(xp-y)^2=p^2(a^2+b^2)+b^2,$$

$$\text{or }y=px\pm\sqrt{[p^2(a^2+b^2)+b^2]}.$$

This is Clairaut's form, so that we have

$$y=mx\pm\sqrt{[m^2(a^2+b^2)+b^2]},$$

which is the well-known tangent to an ellipse.

Also solved by *H. C. WHITAKER*, and *G. B. M. ZERR*.

MECHANICS.

108. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pa.

Can it be shown, as a result of Kepler's third law, that the distances are inversely proportional to the squares of the velocities ?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

This can be demonstrated for circular orbits as follows :

$$t^2:t_1^2=a^3:a_1^3;\text{ but } t^2=4\pi^2a^2/v^2,\ t_1^2=4\pi^2a_1^2/v_1^2.$$

$\therefore\ 1/v^2:1/v_1^2=a:a_1$.

For elliptic orbits

$$v^2=\frac{\mu}{b}\left(\frac{2b-r}{r}\right);\ v_1^2=\frac{\mu}{b_1}\left(\frac{2b_1-R}{R}\right)$$

where b, b_1 are the semi-major axes, respectively.

$$\therefore\ \frac{1}{v^2}:\frac{1}{v_1^2}=\frac{br}{2b-r}:\frac{b_1R}{2b_1-R}=r:R \text{ when } r=R=\infty.$$

$\therefore$ The theorem is true for parabolic orbits.

109. Proposed by **F. P. MATZ, M. Sc., Ph. D.,** Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pa.

If the sun were moved into the center of the earth's orbit, how much would the present length of the year be changed?

Solution by **G. B. M. ZERR, A. M., Ph. D.,** Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

It is presumed in this solution that the earth's orbit remains the same.

Let t=the present length=1 year; T=the required length; μ=absolute force; and a=semi-major axis.

$$\therefore\ t=2\pi\sqrt{\frac{a^3}{\mu}},\ T=\frac{2\pi}{\sqrt{\mu}}.\quad \therefore\ \frac{t}{T}=a\sqrt{a} \text{ or } T=\frac{t}{a\sqrt{a}}.$$

But $a=1$, and $t=1$ year.

$\therefore$ $T=1$ year and the length of the year would remain the same.

110. Proposed by **W. H. DRANE,** Graduate Student, Harvard University, Cambridge, Mass.

A tight roll of very thin and perfectly flexible oilcloth is placed upon a rough inclined plane, a portion of the cloth being unrolled, and, extending from underneath the roll, is spread out smoothly upon the inclined plane below. The roll is then allowed to descend under the action of gravity, picking up the cloth as it goes. Determine the motion as far as possible.

No solution of this problem has been received.

111. Proposed by **WALTER H. DRANE,** Graduate Student, Harvard Univercity, Cambridge, Mass.

A man, weighing 150 pounds at the surface of the earth, ascends in a balloon until the area visible to him is $2\pi R^2(1-\frac{1}{2}\sqrt{2})$. What is his weight at that height?

Solution by **C. HORNUNG, A. M.,** Heidelberg University, Tiffin, O.; **P. S. BERG, B. Sc.,** Larimore, N. D.; **J. SCHEFFER, A. M.,** Hagerstown, Md.; **G. B. M. ZERR, A. M., Ph. D.,** The Temple College, Philadelphia, Pa.

If R=the radius of the earth, and h=the altitude of the zone visible, then

$$2\pi Rh=2\pi R^2(1-\tfrac{1}{2}\sqrt{2}),\text{ whence } h=R(1-\tfrac{1}{2}\sqrt{2}).$$

Now if H be the distance of the balloonist from the earth's center, then